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Hard versus fuzzy c-means clustering for color quantization

Quan Wen[1] and M Emre Celebi[2*]

Abstract

Color quantization is an important operation with many applications in graphics and image processing. Most quantization methods are essentially based on data clustering algorithms. Recent studies have demonstrated the effectiveness of hard c-means (k-means) clustering algorithm in this domain. Other studies reported similar findings pertaining to the fuzzy c-means algorithm. Interestingly, none of these studies directly compared the two types of c-means algorithms. In this study, we implement fast and exact variants of the hard and fuzzy c-means algorithms with several initialization schemes and then compare the resulting quantizers on a diverse set of images. The results demonstrate that fuzzy c-means is significantly slower than hard c-means, and that with respect to output quality, the former algorithm is neither objectively nor subjectively superior to the latter.

1 Introduction

True-color images typically contain thousands of colors, which makes their display, storage, transmission, and processing problematic. For this reason, color quantization (reduction) is commonly used as a preprocessing step for various graphics and image processing tasks. In the past, color quantization was a necessity due to the limitations of the display hardware, which could not handle over 16 million possible colors in 24-bit images. Although 24-bit display hardware has become more common, color quantization still maintains its practical value [1]. Modern applications of color quantization in graphics and image processing include: (i) compression [2], (ii) segmentation [3], (iii) text localization/detection [4], (iv) color-texture analysis [5], (v) watermarking [6], (vi) non-photorealistic rendering [7], (vii) and content-based retrieval [8].

The process of color quantization is mainly comprised of two phases: palette design (the selection of a small set of colors that represents the original image colors) and pixel mapping (the assignment of each input pixel to one of the palette colors). The primary objective is to reduce the number of unique colors, N', in an image to C, $C \ll N'$, with minimal distortion. In most applications, 24-bit pixels in the original image are reduced to 8 bits or fewer. Since natural images often contain a large number of colors, faithful representation of these images with a limited size palette is a difficult problem.

Color quantization methods can be broadly classified into two categories [9]: image-independent methods that determine a universal (fixed) palette without regard to any specific image [10] and image-dependent methods that determine a custom (adaptive) palette based on the color distribution of the images. Despite being very fast, image-independent methods usually give poor results since they do not take into account the image contents. Therefore, most of the studies in the literature consider only image-dependent methods, which strive to achieve a better balance between computational efficiency and visual quality of the quantization output.

Numerous image-dependent color quantization methods have been developed in the past three decades. These can be categorized into two families: preclustering methods and postclustering methods [1]. Preclustering methods are mostly based on the statistical analysis of the color distribution of the images. Divisive preclustering methods start with a single cluster that contains all N' image colors. This initial cluster is recursively subdivided until C clusters are obtained. Well-known divisive methods include median-cut [11], octree [12], variance-based method [13], binary splitting method [14], and greedy orthogonal bipartitioning method [15]. On the other hand, agglomerative preclustering methods [16-18] start with N' singleton clusters each of which

* Correspondence: ecelebi@lsus.edu
[2]Department of Computer Science, Louisiana State University, Shreveport, LA, USA
Full list of author information is available at the end of the article

contains one image color. These clusters are repeatedly merged until C clusters remain. In contrast to preclustering methods that compute the palette only once, postclustering methods first determine an initial palette and then improve it iteratively. Essentially, any data clustering method can be used for this purpose. Since these methods involve iterative or stochastic optimization, they can obtain higher quality results when compared to preclustering methods at the expense of increased computational time. Clustering algorithms adapted to color quantization include hard c-means [19-22], competitive learning [23-27], fuzzy c-means [28-32], and self-organizing maps [33-35].

In this paper, we compare the performance of hard and fuzzy c-means algorithms within the context of color quantization. We implement several efficient variants of both algorithms, each one with a different initialization scheme, and then compare the resulting quantizers on a diverse set of images. The rest of the paper is organized as follows. Section 2 reviews the notions of hard and fuzzy partitions and gives an overview of the hard and fuzzy c-means algorithms. Section 3 describes the experimental setup and compares the hard and fuzzy c-means variants on the test images. Finally, Sect. 4 gives the conclusions.

2 Color quantization using c-means clustering algorithms

2.1 Hard versus fuzzy partitions

Given a data set $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\} \in \mathbb{R}^D$, a real matrix $\mathbf{U} = [u_{ik}]_{C\times N}$ represents a *hard C-partition* of $\mathbf{X}$ if and only if its elements satisfy three conditions [36]:

$$\begin{aligned} & u_{ik} \in \{0,1\} \quad 1 \le i \le C, \quad 1 \le k \le N \\ & \sum_{i=1}^{C} u_{ik} = 1 \quad 1 \le k \le N \\ & 0 < \sum_{k=1}^{N} u_{ik} < N \quad 1 \le i \le C. \end{aligned} \tag{1}$$

Row i of $\mathbf{U}$, say $\mathbf{U}_i = (u_{i1}, u_{i2}, \ldots, u_{iN})$, exhibits the characteristic function of the ith partition (cluster) of $\mathbf{X}$: u_{ik} is 1 if $\mathbf{x}_k$ is in the ith partition and 0 otherwise; $\sum_{i=1}^{C} u_{ik} = 1 \quad \forall k$ means that each $\mathbf{x}_k$ is in exactly one of the C partitions; $0 < \sum_{k=1}^{N} u_{ik} < N \quad \forall i$ means that no partition is empty and no partition is all of $\mathbf{X}$, i.e. $2 \le c \le N$. For obvious reasons, $\mathbf{U}$ is often called a *partition (membership) matrix*.

The concept of *hard C-partition* can be generalized by relaxing the first condition in Equation 1 as $u_{ik} \in 0[1]$ in which case the partition matrix $\mathbf{U}$ is said to represent a *fuzzy C-partition* of $\mathbf{X}$ [37]. In a fuzzy partition matrix $\mathbf{U}$, the total membership of each $\mathbf{x}_k$ is still 1, but since $0 \le u_{ik} \le 1 \ \forall i, k$, it is possible for each $\mathbf{x}_k$ to have an arbitrary distribution of membership among the C fuzzy partitions $\{\mathbf{U}_i\}$.

2.2 Hard c-means (HCM) clustering algorithm

HCM is inarguably one of the most widely used methods for data clustering [38]. It attempts to generate optimal hard C-partitions of $\mathbf{X}$ by minimizing the following objective functional:

$$J(\mathbf{U}, \mathbf{V}) = \sum_{k=1}^{N} \sum_{i=1}^{C} u_{ik} (d_{ik})^2 \tag{2}$$

where $\mathbf{U}$ is a hard partition matrix as defined in §2.1, $\mathbf{V} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_C\} \in \mathbb{R}^D$ is a set of C cluster representatives (centers), e.g. $\mathbf{v}_i$ is the center of hard cluster $\mathbf{U}_i \ \forall i$, and d_{ik} denotes the Euclidean $(\mathcal{L}_2)$ distance between input vector $\mathbf{x}_k$ and cluster center $\mathbf{v}_i$, i.e. $d_{ik} = ||\mathbf{x}_k - \mathbf{v}_i||_2$.

Since $u_{ik} = 1 \Leftrightarrow \mathbf{x}_k \in \mathbf{U}_i$, and is zero otherwise, Equation 2 can also be written as:

$$J(\mathbf{U}, \mathbf{V}) = \sum_{i=1}^{C} \sum_{\mathbf{x}_k \in \mathbf{U}_i} (d_{ik})^2.$$

This problem is known to be NP-hard even for $C = 2$ [39] or $D = 2$ [40], but a heuristic method developed by Lloyd [41] offers a simple solution. Lloyd's algorithm starts with C arbitrary centers, typically chosen uniformly at random from the data points. Each point is then assigned to the nearest center, and each center is recalculated as the mean of all points assigned to it. These two steps are repeated until a predefined termination criterion is met.

The complexity of HCM is $\mathcal{O}(NC)$ per iteration for a fixed D value. In color quantization applications, D often equals three since the clustering procedure is usually performed in a three-dimensional color space such as RGB or CIEL * a * b * [42].

From a clustering perspective, HCM has the following advantages:

◊ It is conceptually simple, versatile, and easy to implement.

◊ It has a time complexity that is linear in N and C.

◊ It is guaranteed to terminate [43] with a quadratic convergence rate [44].

Due to its gradient descent nature, HCM often converges to a local minimum of its objective functional [43] and its output is highly sensitive to the selection of the initial cluster centers. Adverse effects of improper initialization include empty clusters, slower convergence, and a higher chance of getting stuck in bad local minima. From a color quantization perspective, HCM has two additional drawbacks. First, despite its linear time complexity, the iterative nature of the algorithm renders the palette generation phase computationally expensive. Second, the pixel mapping phase is inefficient, since for each input pixel a full search of the palette is required to determine the nearest color. In contrast, preclustering methods often manipulate and store the palette in a special data structure (binary trees are commonly used), which allows for fast nearest neighbor search during the mapping phase. Note that these drawbacks are shared by the majority of postclustering methods, including the fuzzy c-means algorithm.

We have recently proposed a fast and exact HCM variant called Weighted Sort-Means (WSM) that utilizes data reduction and accelerated nearest neighbor search [21,22]. When initialized with a suitable preclustering method, WSM has been shown to outperform a large number of classic and state-of-the-art quantization methods including median-cut [11], octree [12], variance-based method [13], binary splitting method [14], greedy orthogonal bipartitioning method [15], neuquant [33], split and merge method [18], adaptive distributing units method [23,26], finite-state HCM method [19], and stable-flags HCM method [20].

In this study, WSM is used in place of HCM since both algorithms give numerically identical results. However, in the remainder of this paper, WSM will be referred to as HCM for reasons of uniformity.

2.3 Fuzzy c-means (FCM) clustering algorithm

FCM is a generalization of HCM in which points can belong to more than one cluster [36]. It attempts to generate optimal fuzzy C-partitions of **X** by minimizing the following objective functional:

$$J_m(\mathbf{U},\mathbf{V}) = \sum_{k=1}^{N}\sum_{i=1}^{C}(u_{ik})^m(d_{ik})^2 \tag{3}$$

where the parameter $1 \leq m < \infty$ controls the degree of membership sharing between fuzzy clusters in **X**.

As in the case of HCM, FCM is based on an alternating minimization procedure [45]. At each iteration, the fuzzy partition matrix **U** is updated by

$$u_{ik} = \left[\sum_{j=1}^{C}\left(\frac{d_{ik}}{d_{jk}}\right)^{2/(m-1)}\right]^{-1}. \tag{4}$$

which is followed by the update of the prototype matrix **V** by

$$\mathbf{v}_i = \left(\sum_{k=1}^{N}(u_{ik})^m \mathbf{x}_k\right) \Big/ \left(\sum_{k=1}^{N}(u_{ik})^m\right). \tag{5}$$

As $m \xrightarrow{+} 1$, FCM converges to an HCM solution. Conversely, as $m \to \infty$ it can be shown that $u_{ik} \to 1/C$ $\forall i, k$, so $\mathbf{v}_i \to \bar{\mathbf{X}}$, the centroid of **X**. In general, the larger m is, the fuzzier are the membership assignments; and conversely, as $m \xrightarrow{+} 1$, FCM solutions become hard. In color quantization applications, in order to map each input color to the nearest (most similar) palette color, the membership values should be defuzzified upon convergence as follows:

$$\hat{u}_{ik} = \begin{cases} 1 & u_{ik} = \max\limits_{1 \leq j \leq C} u_{jk} \\ 0 & \text{otherwise} \end{cases}.$$

A näive implementation of FCM has a complexity of $\mathcal{O}(NC^2)$ per iteration, which is quadratic in the number of clusters. In this study, a linear complexity formulation, i.e. $\mathcal{O}(NC)$, described in [46] is used. In order to take advantage of the peculiarities of color image data (presence of duplicate samples, limited range, and sparsity), the same data reduction strategy used in WSM is incorporated into FCM.

3 Experimental results and discussion

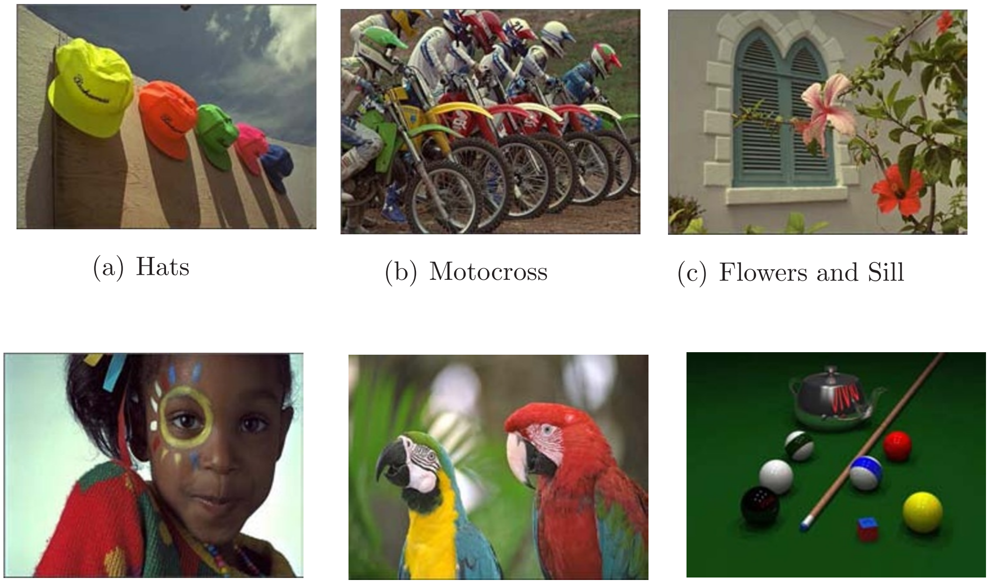
3.1 Image set and performance criteria

Six publicly available, true-color images were used in the experiments. Five of these were natural images from the *Kodak Lossless True Color Image Suite* [47]: Hats (768 × 512; 34,871 unique colors), Motocross (768 × 512; 63,558 unique colors), Flowers and Sill (768 × 512; 37,552 unique colors), Cover Girl (768 × 512; 44,576 unique colors), and Parrots (768 × 512; 72,079 unique colors). The sixth image was synthetic, Poolballs (510 × 383; 13,604 unique colors) [48]. The images are shown in Figure 1.

The effectiveness of a quantization method was quantified by the commonly used mean absolute error (MAE) and mean squared error (MSE) measures:

$$\begin{aligned} \mathrm{MAE}\left(\mathbf{I},\hat{\mathbf{I}}\right) &= \frac{1}{HW}\sum_{h=1}^{H}\sum_{w=1}^{W}\left\|\mathbf{I}(h,w)-\hat{\mathbf{I}}(h,w)\right\|_1 \\ \mathrm{MSE}\left(\mathbf{I},\hat{\mathbf{I}}\right) &= \frac{1}{HW}\sum_{h=1}^{H}\sum_{w=1}^{W}\left\|\mathbf{I}(h,w)-\hat{\mathbf{I}}(h,w)\right\|_2^2 \end{aligned} \tag{6}$$

where **I** and $\hat{\mathbf{I}}$ denote, respectively, the $H \times W$ original and quantized images in the RGB color space. MAE and MSE represent the average color distortion with respect to the $\mathcal{L}_1$ (City-block) and $\mathcal{L}_2^2$ (squared Euclidean) norms, respectively. Note that most of the other popular evaluation measures in the color quantization literature such as peak signal-to-noise ratio (PSNR), normalized



(a) Hats (b) Motocross (c) Flowers and Sill

(d) Cover Girl (e) Parrots (f) Poolballs

Figure 1 Test images. **a** Hats, **b** Motocross, **c** Flowers and Sill, **d** Cover Girl, **e** Parrots, **f** Poolballs.

MSE, root MSE, and average color distortion [24,34] are variants of MAE or MSE.

The efficiency of a quantization method was measured by CPU time in milliseconds, which includes the time required for both the palette generation and the pixel mapping phases. The fast pixel mapping algorithm described in [49] was used in the experiments. All of the programs were implemented in the C language, compiled with the gcc v4.4.3 compiler, and executed on an Intel Xeon E5520 2.26 GHz machine. The time figures were averaged over 20 runs.

3.2 Comparison of HCM and FCM

The following well-known preclustering methods were used in the experiments:

- **Median-cut (MC)** [11]: This method starts by building a 32 × 32 × 32 color histogram that contains the original pixel values reduced to 5 bits per channel by uniform quantization (bit-cutting). This histogram volume is then recursively split into smaller boxes until C boxes are obtained. At each step, the box that contains the largest number of pixels is split along the longest axis at the median point, so that the resulting sub-boxes each contain approximately the same number of pixels. The centroids of the final C boxes are taken as the color palette.
- **Octree (OCT)** [12]: This two-phase method first builds an octree (a tree data structure in which each internal node has up to eight children) that represents the color distribution of the input image and then, starting from the bottom of the tree, prunes the tree by merging its nodes until C colors are obtained. In the experiments, the tree depth was limited to 6.
- **Variance-based method (WAN)** [13]: This method is similar to MC with the exception that at each step the box with the largest weighted variance (squared error) is split along the major (principal) axis at the point that minimizes the marginal squared error.
- **Greedy orthogonal bipartitioning method (WU)** [15]: This method is similar to WAN with the exception that at each step the box with the largest weighted variance is split along the axis that minimizes the sum of the variances on both sides.

Four variants of HCM/FCM, each one initialized with a different preclustering method, were tested. Each variant was executed until it converged. Convergence was determined by the following commonly used criterion [50]: $(J_{(i-1)} - J_{(i)})/J_{(i)} \leq \varepsilon$, where $J_{(i)}$ denotes the value of the objective functional (Eqs. (2) and (3) for HCM and FCM, respectively) at the end of the ith iteration. The convergence threshold was set to $\varepsilon = 0.001$.

The weighting exponent (m) value recommended for color quantization applications ranges between 1.3 [30] and 2.0 [31]. In the experiments, four different m values were tested for each of the FCM variants: 1.25, 1.50, 1.75, and 2.00.

Table 1 MAE comparison of the quantization methods

		Hats						**Motocross**					
			HCM	**FCM**					**HCM**	**FCM**			
C		**Init**		**1.25**	**1.50**	**1.75**	**2.00**	**Init**		**1.25**	**1.50**	**1.75**	**2.00**
32	MC	30	16	16	16	16	15	26	19	19	19	18	18
	OCT	19	15	15	15	15	15	21	17	18	18	18	18
	WAN	26	15	15	15	15	15	24	18	18	18	18	18
	WU	18	15	15	15	15	15	21	18	18	17	17	18
64	MC	18	12	12	11	11	11	20	15	15	14	14	14
	OCT	13	10	10	10	10	10	15	13	13	13	13	13
	WAN	18	11	11	10	10	11	19	14	14	13	13	14
	WU	12	10	10	10	10	10	15	13	13	13	13	13
128	MC	13	9	8	8	8	8	16	12	11	11	11	11
	OCT	9	7	7	7	7	7	12	10	10	10	10	10
	WAN	11	8	7	7	7	7	15	10	10	10	10	11
	WU	9	7	7	7	7	7	12	10	10	10	10	10
256	MC	10	7	6	6	6	6	13	9	9	9	8	9
	OCT	6	5	5	5	5	5	9	8	8	8	8	8
	WAN	9	5	5	5	5	5	12	8	8	8	8	8
	WU	6	5	5	5	5	5	9	8	8	8	8	8
		Flowers and Sill						Cover Girl					
			HCM	FCM					HCM	FCM			
C		Init		1.25	1.50	1.75	2.00	Init		1.25	1.50	1.75	2.00
32	MC	20	14	14	14	13	13	22	16	15	14	14	14
	OCT	15	12	12	12	12	12	17	14	14	14	13	13
	WAN	17	12	12	12	12	12	18	14	14	14	14	14
	WU	14	12	12	12	12	12	16	14	14	14	14	14
64	MC	14	11	10	10	10	10	16	11	11	11	11	10
	OCT	11	9	9	9	9	9	12	10	10	10	10	10
	WAN	12	9	9	9	9	9	15	11	11	10	10	11
	WU	10	9	9	9	9	9	12	10	10	10	10	10
128	MC	12	8	8	8	7	7	13	9	8	8	8	8
	OCT	8	7	7	7	7	7	9	8	7	7	7	8
	WAN	9	7	7	7	7	7	12	8	8	8	8	8
	WU	8	7	7	7	7	7	9	8	8	8	8	8
256	MC	9	6	6	6	6	6	11	7	7	6	6	6
	OCT	6	5	5	5	5	5	7	6	6	6	6	6
	WAN	8	5	5	5	5	5	10	6	6	6	6	6
	WU	6	5	5	5	5	5	7	6	6	6	6	6
		Parrots						Poolballs					
			HCM	FCM					HCM	FCM			
C		Init		1.25	1.50	1.75	2.00	Init		1.25	1.50	1.75	2.00
32	MC	28	21	21	20	21	21	12	9	9	9	7	7
	OCT	24	20	20	20	20	20	8	6	6	6	6	6
	WAN	25	21	20	20	20	20	11	6	6	6	6	6
	WU	23	20	20	20	20	20	7	7	6	6	6	6
64	MC	22	15	15	15	15	15	9	6	6	6	5	5
	OCT	18	15	15	15	15	15	5	4	4	3	3	4
	WAN	19	15	15	15	15	15	9	4	4	4	4	4
	WU	17	15	15	15	15	15	5	4	4	4	4	4
128	MC	16	12	12	12	12	12	7	5	5	5	4	3

Table 1 MAE comparison of the quantization methods *(Continued)*

	OCT	14	11	11	11	11	11	3	2	2	2	2	2
	WAN	15	11	11	11	11	12	9	3	3	3	3	3
	WU	13	11	11	11	11	11	4	3	3	3	2	2
256	MC	13	9	9	9	9	9	7	4	3	3	3	2
	OCT	10	9	8	8	9	9	2	2	2	2	2	2
	WAN	12	9	9	9	9	9	8	2	2	2	2	2
	WU	10	9	8	8	9	9	4	2	2	2	2	2

Table 2 MSE comparison of the quantization methods

		Hats						**Motocross**					
			HCM	**FCM**					**HCM**	**FCM**			
C		**Init**		**1.25**	**1.50**	**1.75**	**2.00**	**Init**		**1.25**	**1.50**	**1.75**	**2.00**
32	MC	618	159	169	163	175	185	427	217	209	229	236	253
	OCT	293	185	184	187	214	242	301	197	203	249	277	280
	WAN	624	162	160	165	172	201	446	194	193	220	235	291
	WU	213	157	157	156	163	172	268	191	191	194	198	208
64	MC	192	91	87	86	87	99	232	125	123	119	125	134
	OCT	132	79	79	78	87	94	159	111	112	122	129	142
	WAN	311	89	83	84	100	110	292	112	111	117	122	141
	WU	103	72	75	75	79	85	147	109	109	111	121	126
128	MC	111	47	45	45	50	52	154	76	74	72	75	86
	OCT	65	43	43	43	48	52	96	65	65	69	76	91
	WAN	106	44	42	44	48	51	169	66	66	68	72	85
	WU	52	38	40	40	42	46	87	63	63	65	70	84
256	MC	63	29	27	26	28	31	100	49	45	45	48	57
	OCT	34	22	24	25	28	33	54	39	39	42	48	55
	WAN	53	21	23	24	26	30	92	39	39	40	44	53
	WU	30	21	23	23	25	28	51	38	38	39	43	50
		Flowers and Sill						Cover Girl					
			HCM	FCM					HCM	FCM			
C		Init		1.25	1.50	1.75	2.00	Init		1.25	1.50	1.75	2.00
32	MC	257	117	117	114	112	120	269	142	132	127	130	135
	OCT	155	102	102	102	109	120	182	127	127	128	131	137
	WAN	198	102	100	101	107	114	230	126	127	129	133	137
	WU	134	101	100	101	103	108	162	126	125	126	129	133
64	MC	113	66	64	64	65	70	145	79	78	76	80	85
	OCT	88	58	57	58	66	75	105	72	72	75	78	87
	WAN	98	56	55	56	59	64	157	75	75	77	83	88
	WU	71	53	56	57	59	61	93	71	72	73	76	82
128	MC	84	42	39	38	39	43	104	52	45	44	47	56
	OCT	47	33	33	34	37	42	62	42	42	44	47	52
	WAN	57	29	32	33	35	39	102	44	43	45	50	57
	WU	40	30	32	32	34	38	55	41	40	41	44	49
256	MC	48	23	24	23	24	27	68	32	29	28	29	34
	OCT	26	19	21	21	24	27	36	25	25	25	29	33
	WAN	37	18	20	20	22	25	63	26	25	26	28	32
	WU	26	18	20	20	22	24	33	24	24	24	26	31
		Parrots						Poolballs					
			HCM	FCM					HCM	FCM			
C		Init		1.25	1.50	1.75	2.00	Init		1.25	1.50	1.75	2.00

Table 2 MSE comparison of the quantization methods *(Continued)*

32	MC	418	240	240	241	274	285	136	74	72	71	66	61
	OCT	342	247	246	246	255	265	130	74	67	75	85	88
	WAN	376	246	239	246	254	263	112	49	49	50	52	54
	WU	299	234	234	237	244	256	68	50	50	50	50	54
64	MC	274	137	137	138	140	157	64	39	39	39	28	30
	OCT	191	133	132	135	140	155	48	29	27	28	29	34
	WAN	233	131	131	132	141	164	59	22	22	22	22	24
	WU	167	130	130	131	135	155	31	22	21	21	22	23
128	MC	147	82	80	82	86	95	38	22	21	19	15	15
	OCT	111	79	78	79	85	97	20	12	12	12	13	16
	WAN	153	78	77	80	88	97	45	12	11	11	11	12
	WU	95	77	77	78	83	91	17	11	10	10	11	11
256	MC	96	50	49	49	53	62	27	13	10	9	8	8
	OCT	64	48	47	50	54	61	9	6	5	6	6	7
	WAN	92	44	47	49	55	61	38	6	6	5	6	6
	WU	58	46	46	48	52	59	11	6	5	5	6	6

Table 3 CPU time comparison of the quantization methods

		Hats					**Motocross**				
		HCM	**FCM**				**HCM**	**FCM**			
C			**1.25**	**1.50**	**1.75**	**2.00**		**1.25**	**1.50**	**1.75**	**2.00**
32	MC	48	2,664	3,238	3,192	934	84	11,797	7,749	9,244	1,895
	OCT	80	1,883	2,032	1,656	691	110	4,139	5,034	4,054	912
	WAN	45	3,406	2,709	2,980	762	60	4,261	2,971	4,013	715
	WU	50	1,976	2,227	1,854	425	60	4,547	4,751	4,016	974
64	MC	59	10,536	11,059	5,494	1,211	101	29,081	24,021	24,858	5,640
	OCT	97	5,045	7,353	5,533	1,379	130	10,154	8,752	9,366	1,857
	WAN	62	9,350	9,729	10,303	1,501	94	12,531	8,842	10,308	3,160
	WU	54	4,228	4,756	4,822	1,332	71	6,361	6,903	8,441	2,020
128	MC	108	20,269	19,945	15,815	2,879	156	49,930	54,102	57,146	14,704
	OCT	141	12,700	11,745	8,799	2,444	180	22,410	20,504	18,866	5,297
	WAN	89	22,871	13,143	11,544	2,071	125	17,472	19,467	23,061	5,683
	WU	76	12,719	11,191	11,114	2,300	113	15,604	14,833	13,684	5,049
256	MC	267	42,670	51,559	35,602	6,126	607	144,758	116,915	131,130	28,752
	OCT	306	20,287	19,512	17,806	5,039	328	39,101	42,906	37,946	7,988
	WAN	202	26,505	20,574	18,794	5,649	380	50,621	45,127	38,105	9,152
	WU	191	19,058	20,692	18,763	5,434	284	39,098	43,176	32,835	8,767
		Flowers and Sill					Cover Girl				
		HCM	FCM				HCM	FCM			
C			1.25	1.50	1.75	2.00		1.25	1.50	1.75	2.00
32	MC	56	5,591	5,633	5,243	1,385	55	6,067	6,772	7,402	1,545
	OCT	81	2,618	4,151	3,447	645	82	1,992	2,615	2,026	584
	WAN	42	2,240	2,525	2,625	709	45	1,934	1,988	1,975	613
	WU	42	2,111	1,585	1,590	547	41	1,927	1,692	2,264	511
64	MC	62	10,508	9,098	8,938	1,970	77	14,165	24,945	18,248	4,979
	OCT	99	9,091	6,579	7,396	1,369	100	6,431	6,775	4,570	1,803
	WAN	58	5,413	4,060	4,491	1,067	59	6,540	9,785	7,905	2,574
	WU	53	3,887	3,992	3,434	1,005	62	5,745	4,913	4,242	1,409
128	MC	124	35,372	31,854	28,658	4,198	120	47,186	45,248	34,731	9,428

Table 3 CPU time comparison of the quantization methods *(Continued)*

	OCT	120	9,787	11,505	11,709	2,375	130	12,311	13,002	9,794	2,290
	WAN	86	10,875	10,344	11,189	2,378	103	19,432	12,332	13,069	3,347
	WU	84	9,145	12,170	9,570	2,897	95	11,016	9,889	8,602	2,872
256	MC	368	63,209	64,305	46,177	9,147	403	84,079	104,289	71,327	19,082
	OCT	291	30,560	27,794	23,475	4,738	279	31,042	27,404	25,272	6,417
	WAN	223	28,113	21,109	33,265	5,994	238	33,780	31,421	35,709	6,883
	WU	226	19,480	19,660	19,310	5,480	216	27,107	25,100	26,488	7,728
		Parrots					Poolballs				
		HCM	FCM				HCM	FCM			
C			1.25	1.50	1.75	2.00		1.25	1.50	1.75	2.00
32	MC	74	8,209	9,359	6,894	1,917	15	1,076	813	1,004	518
	OCT	124	8,127	8,586	13,018	2,408	31	980	1,041	974	305
	WAN	65	8,465	4,977	4,095	1,172	15	549	467	441	116
	WU	60	3,793	3,346	3,071	1,362	15	729	1,080	1,274	201
64	MC	120	16,492	16,168	18,400	4,936	17	1,556	1,504	2,819	708
	OCT	132	10,659	8,395	9,286	2,773	36	3,261	2,625	2,692	519
	WAN	85	11,756	12,993	8,709	3,065	19	1,133	1,396	1,103	371
	WU	80	6,438	6,155	6,665	2,184	20	1,353	1,056	867	314
128	MC	158	49,581	49,913	42,309	12,247	33	2,492	5,939	4,760	849
	OCT	181	28,474	27,161	26,921	5,902	51	3,032	2,385	3,310	1,042
	WAN	136	30,827	20,314	23,764	6,878	36	3,576	4,150	2,517	767
	WU	122	15,272	19,182	20,661	6,875	33	4,816	3,629	3,484	581
256	MC	536	128,094	103,153	104,613	20,178	224	15,378	10,863	9,566	2,499
	OCT	391	54,419	57,325	41,750	10,665	144	6,091	6,194	5,398	1,306
	WAN	380	63,969	59,283	50,189	16,601	120	6,372	4,831	6,123	1,292
	WU	306	42,535	38,776	43,910	12,148	113	4,977	5,865	7,330	1,291

Tables 1 and 2 compare the effectiveness of the HCM and FCM variants on the test images. Similarly, Table 3 gives the efficiency comparison. For a given number of colors C ($C \in \{32, 64, 128, 256\}$), preclustering method P($P \in \{\text{MC, OCT, WAN, WU}\}$), and input image $\mathbf{I}$, the column labeled as 'Init' contains the MAE/MSE between $\mathbf{I}$ and $\hat{\mathbf{I}}$ (the output image obtained by reducing the number of colors in $\mathbf{I}$ to C using P), whereas the one labeled as 'HCM' contains the MAE/MSE value obtained by HCM when initialized by P. The remaining four columns contain the MAE/MSE values obtained by the FCM variants. Note that HCM is equivalent to FCM with m = 1.00. The following observations are in order (note that each of these comparisons is made within the context of a particular C, P, and $\mathbf{I}$ combination):

▹ The most effective initialization method is WU, whereas the least effective one is MC.

▹ Both HCM and FCM reduces the quantization distortion regardless of the initialization method used. However, the percentage of MAE/MSE reduction is more significant for some initialization methods than others. In general, HCM/FCM is more likely to obtain a significant improvement in MAE/MSE when initialized by an ineffective preclustering algorithm such as MC or WAN. This is not surprising given that such ineffective methods generate outputs that are likely to be far from a local minimum, and hence HCM/FCM can significantly improve upon their results.

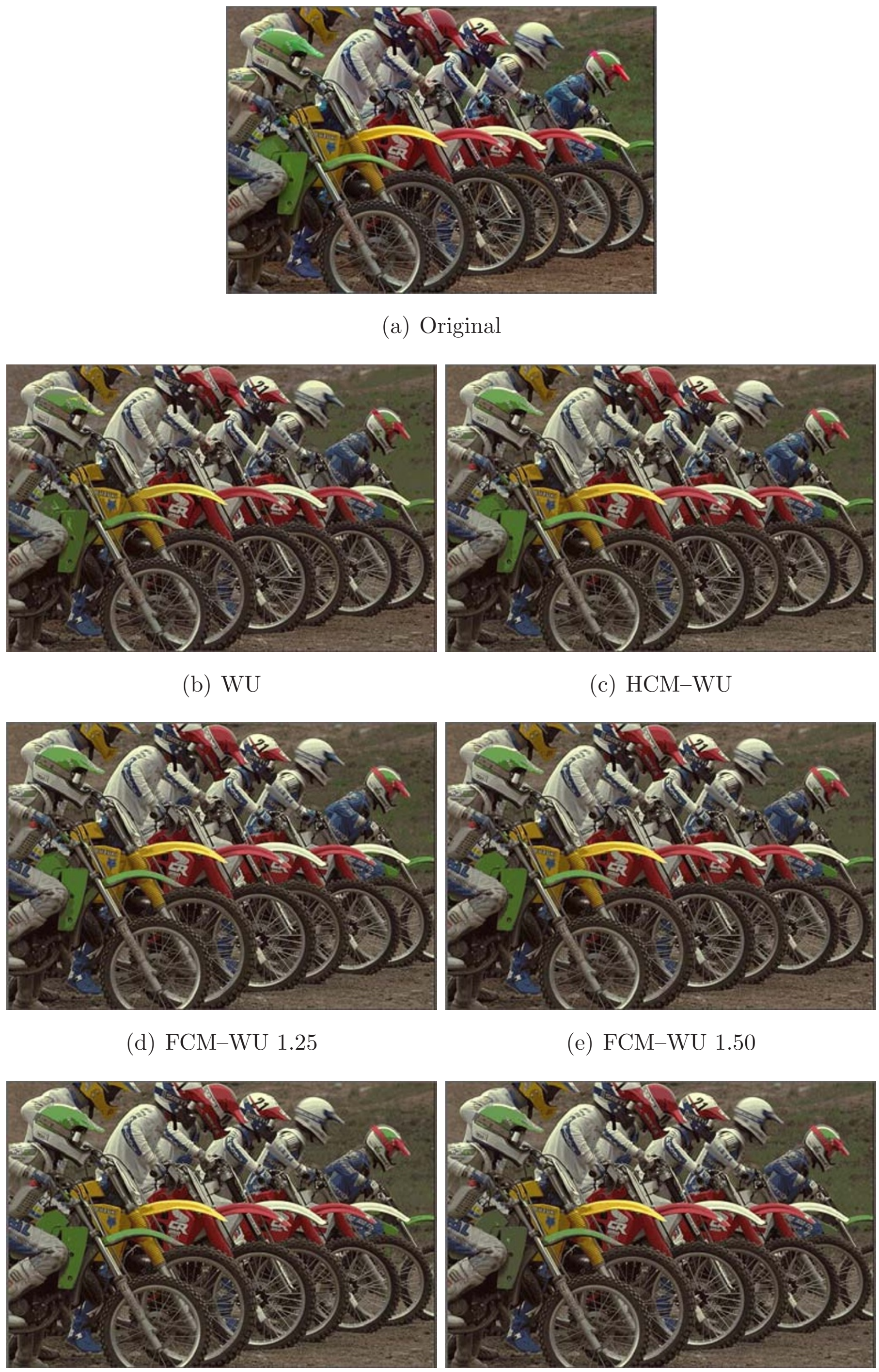
▹ With respect to MAE, the HCM variant and the four FCM variants have virtually identical performance.

▹ With respect to MSE, the performances of the HCM variant and the FCM variant with m = 1.25 are indistinguishable. Furthermore, the effectiveness of the FCM variants degrades with increasing m value.

▹ On average, HCM is 92 times faster than FCM. This is because HCM uses hard memberships, which makes possible various computational optimizations that do not affect accuracy of the algorithm [51-55]. On the other hand, due to the intensive fuzzy membership calculations involved, accelerating FCM is significantly more difficult, which is why the majority of existing acceleration methods involve approximations [56-60]. Note that the fast HCM/FCM implementations used in this study give exactly the same results as the conventional HCM/FCM.

▹ The FCM variant with m = 2.00 is the fastest since, among the m values tested in this study, only m = 2.00 leads to integer exponents in Equations 4 and 5.

Figure 2 shows sample quantization results for the Motocross image. Since WU is the most effective initialization method, only the outputs of HCM/FCM variants that use WU are shown. It can be seen that WU is



(a) Original

(b) WU

(c) HCM–WU

(d) FCM–WU 1.25

(e) FCM–WU 1.50

(f) FCM–WU 1.75

(g) FCM–WU 2.00

Figure 2 Sample quantization results for the Motocross image (*C* = 32). **a** Original, **b** WU, **c** HCM-WU, **d** FCM-WU 1.25, **e** FCM-WU 1.50, **f** FCM-WU 1.75, **g** FCM-WU 2.00.

unable to represent the color distribution of certain regions of the image (fenders of the leftmost and rightmost dirt bikes, helmet of the driver of the leftmost dirt bike, grass, etc.) In contrast, the HCM/FCM variants perform significantly better in allocating representative colors to these regions. Note that among the FCM variants, the one with m = 2.00 performs slightly worse in that the body color of the leftmost dirk bike and the color of the grass are mixed.

Figure 3 shows sample quantization for the Hats image. It can be seen that WU causes significant contouring in the sky region. It also adds a red tint to the

(a) Original

(b) WU

(c) HCM–WU

(d) FCM–WU 1.25

(e) FCM–WU 1.50

(f) FCM–WU 1.75

(g) FCM–WU 2.00

Figure 3 Sample quantization results for the Hats image (*C* = 64). **a** Original, **b** WU, **c** HCM-WU, **d** FCM-WU 1.25, **e** FCM-WU 1.50, **f** FCM-WU 1.75, **g** FCM-WU 2.00.

pink hat. On the other hand, the HCM/FCM variants are significantly better in representing these regions. Once again, the less fuzzy FCM variants, i.e. those with smaller m values, are slightly better than the more fuzzy ones. For example, in the outputs of FCM 1.75 and 2.00, a brownish region can be discerned in the upper-right region where the white cloud and the blue sky merge.

It could be argued that HCM's objective functional, Equation 2, is essentially equivalent to MSE, Equation 6, and therefore it is unreasonable to expect FCM to outperform HCM with respect to MSE unless $m \approx 1.00$. However, neither HCM nor FCM minimizes MAE and yet their MAE performances are nearly identical. Hence, it can be safely concluded that FCM is not superior to HCM with respect to quantization effectiveness. Moreover, due to its simple formulation, HCM is amenable to various optimization techniques, whereas FCM's formulation permits only modest acceleration. Therefore, HCM should definitely be preferred over FCM when computationally efficiency is of prime importance.

4 Conclusions

In this paper, hard and fuzzy c-means clustering algorithms were compared within the context of color quantization. Fast and exact variants of both algorithms with several initialization schemes were compared on a diverse set of publicly available test images. The results indicate that fuzzy c-means does not seem to offer any advantage over hard c-means. Furthermore, due to the intensive membership calculations involved, fuzzy c-means is significantly slower than hard c-means, which makes it unsuitable for time-critical applications. In contrast, as was also demonstrated in a recent study [22], an efficient implementation of hard c-means with an appropriate initialization scheme can serve as a fast and effective color quantizer.

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Author details

[1]School of Computer Science and Engineering, University of Electronic Science and Technology of China, Chengdu, People's Republic of China
[2]Department of Computer Science, Louisiana State University, Shreveport, LA, USA

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The authors declare that they have no competing interests.

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